

Exam. Code : 224802  
Subject Code : 104776

M.Sc. (Physics) 2<sup>nd</sup> Semester (Batch 2023-25) (CBGS)  
PHY-451 : QUANTUM MECHANICS-I

Time Allowed—3 Hours] [Maximum Marks—100

**Note** :— Attempt FIVE questions in all, selecting at least ONE question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

#### SECTION—A

- (a) What is Hilbert space and linear vector space ? Discuss dimension and basis of a vector space. Define ket and bra vector along with its properties. Discuss difference between observables and operators. 15  
(b) Let  $|\varphi_1\rangle$  and  $|\varphi_2\rangle$  be two orthonormal state vectors. Let  $A = |\varphi_1\rangle\langle\varphi_2| + |\varphi_2\rangle\langle\varphi_1|$ . Is A a projection operator ? 5
- (a) What is Unitary transformations ? Discuss its properties . Also, explain the infinitesimal and finite Unitary transformations in detail. 15  
(b) Consider the state vector  $|\psi\rangle = \begin{pmatrix} 5i \\ 2 \\ -i \end{pmatrix}$  find  $|\psi\rangle^*$ . 5

14101(2524)/IK-2169

1

(Contd.)

<https://www.gnduonline.com>

#### SECTION—B

- (a) What do you mean by time evolution operator ? Discuss the time evolution of the system's state expectation values. 10  
(b) Describe the momentum operator in the position representation and position operator in the momentum representation. 10
- (a) State and prove Ehrenfest's theorem and show how it leads to the correspondence principle. 10  
(b) Obtain the equations of motion for the state vectors and operators in Heisenberg picture. 10

#### SECTION—C

- (a) Solve the Schrödinger's equation for the linear harmonic oscillator and obtain the energy Eigen values. 15  
(b) Find the expectation value of the energy when the state of the oscillator is described by the following wave function  $\psi(x, t) = \frac{1}{\sqrt{2}}[\psi_0(x, t) + \psi_1(x, t)]$ , where  $\psi_0(x, t)$  and  $\psi_1(x, t)$  are the wave-functions for the ground state and the first excited state respectively. 5

<https://www.gnduonline.com>

6. A particle travelling with energy  $E$  along  $x$ -axis has in its path a rectangular potential barrier of height  $V < E$  and width  $a$ . Calculate the transmission coefficient of the particle and discuss briefly its application to the observed phenomenon of  $\alpha$ -decay in nuclei. 20

SECTION—D

7. Obtain Schrödinger's equation for spherically symmetric case of three dimensional harmonic oscillator. Solve it to obtain Eigen functions and Eigen values. 20
8. (a) The electron of the hydrogen atom is in its ground state. Determine  $\langle r \rangle$ ,  $\langle r^2 \rangle$  and the most probable value of  $r$  for this case. 10
- (b) If  $J_x, J_y, J_z$  are angular momentum operators, show that  $[J^2, J_{\pm}] = 0, [J_z, J_{\pm}] = \pm \hbar J_{\pm}, [J_+, J_-] = 2\hbar J_z$  where,  $J_+ = J_x + iJ_y$  and  $J_- = J_x - iJ_y$ . 10