

Exam. Code : 209001
Subject Code : 4763

M.Sc. Physics 1st Semester
MATHEMATICAL PHYSICS
Paper—PHY-402

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Candidates are to attempt **FIVE** questions,
ONE from each Section. **Fifth** question may be
attempted from any Section. All questions carry
equal marks.

SECTION—A

1. (a) State and prove convolution theorem for Fourier Transform. What does physically it represent ?
10

- (b) Find the Fourier Transform of :

$$f(x) = \begin{cases} 1 - |x| & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$$

Hence prove that :

$$\int_0^{\infty} \left(\frac{\sin t}{t} \right)^4 dt = \frac{\pi}{3}. \quad \text{10}$$

2. (a) Construct a scalar from the tensor A_{kl}^{ij} .
5
- (b) If $ds^2 = g_{ij} dx^i dx^j$ is invariant, show that g_{ij} is a symmetric covariant tensor of second rank.
5
- (c) Using the operator formalism of orbital angular momentum, prove that :

$$\vec{L} \times \vec{L} = i\vec{L}. \quad \text{10}$$

SECTION—B

3. The interaction of two particles is described by a potential $V(x) = A \frac{e^{-ax}}{x}$, where A is negative constant.
Solve the following resultant Schrodinger equation :

$$\frac{\hbar^2}{2\mu} \frac{d^2y}{dx^2} + (E - V)y = 0. \quad \text{20}$$

4. (a) Write down an expression for the generating function of Bessel function, J_n . Use it to prove that $\frac{dJ_n}{dx} = \frac{1}{2}(J_{n-1} + J_{n+1})$ for the case of an integer n .
10
- (b) Define Gamma function, $\Gamma(n)$, show that :

$$\Gamma\left(\frac{1}{2} - n\right) \Gamma\left(\frac{1}{2} + n\right) = (-1)^n \pi. \quad \text{10}$$

SECTION—C

5. (a) Find the analytic function $w(z) = u(x, y) + iv(x, y)$,
 (i) if $u(x, y) = x^3 - 3xy^2$, (ii) if $v(x, y) = e^{-y} \sin x$.

10

- (b) State and prove Cauchy residue theorem. Hence evaluate the integral $\int_0^{2\pi} \frac{d\theta}{3 - 2\cos\theta + \sin\theta}$. 10

6. (a) If $f(z)$ is a real function of the complex variable $z = x + iy$ and the Laurent expansion about the origin, $f(z) = \sum a_n z^n$, has $a_n = 0$ for $n < -N$, show that all the coefficients of a_n are real. 10

- (b) Show that $\int_{z_0}^z Z^n dz = \frac{z^{n+1} - z_0^{n+1}}{n+1}$ for all n , except $n = -1$. What physically $n = -1$ represent ? 10

SECTION—D

7. (a) Define permutation group and give one example. Discuss their importance in quantum mechanics of identical particles. 10

- (b) Define the Unitary matrices and show that unitary matrices of order n form a group under matrix multiplication. 10

8. (a) Explain with at least two examples what are the differences between isomorphism and homomorphism. 10

- (b) Define $SO(2)$ and $SO(3)$ are the rotational groups. What are the basic differences between these groups ? 10