Exam. Code : 103205
Subject Code : 1216

B.A./B.Sc. Semester—V

MATHEMATICS

(Vector Calculus & Solid Geometry)

Paper—I

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt any **FIVE** questions in all, choosing at least **TWO** from each Section.

SECTION—A

I. (a) Define a vector function. Discuss geometrical interpretations of position vector $\vec{r}$ and $\frac{d\vec{r}}{dt}$.

(b) If $\vec{r} \cdot d\vec{r} = 0$, then show that $\vec{r}$ is a constant. 5,5

II. (a) Find the directional derivative of $f(x, y, z) = x^3y^3z^3$ at the point (1, 1, –1) in the direction of tangent to the curve $x = e^t$, $y = 2 \sin t + 1$, $z = t - \cos t$ at $t = 0$.

(b) Determine a constant l so that the vector $\vec{F} = (x + 3y)i + (y - 2z)j + (x + lz)k$ is solenoidal.

(c) Define gradient of a scaler point function. Show that $\nabla[\vec{r}, \vec{a}, \vec{b}] = \vec{a} \times \vec{b}$, where $\vec{a}$ and $\vec{b}$ are constant vectors. 4,2,4

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III. (a) Define curl of a vector point function and discuss its physical interpretation.

(b) Prove that :

$$\text{grad}(\vec{u}.\vec{v}) = \vec{v}.\nabla\vec{u} + \vec{u}.\nabla\vec{v} + \vec{v} \times \text{curl}\,\vec{u} + \vec{u} \times \text{curl}\,\vec{v}.$$

5,5

IV. (a) State and prove Stoke's theorem.

(b) Prove that $\text{div curl}\,\vec{f} = 0$, where $\vec{f}$ in any continuously differentiable vector point function. 8,2

V. (a) Prove that $r^n\vec{r}$ is irrotational. Find n when it is solenoidal.

(b) Find the circulation of $\vec{F}$ round the curve C, where $\vec{F} = (2x + y^2)\vec{i} + (3y - 4x)\vec{j}$ and C is the curve $y = x^2$ from (0, 0) to (1, 1) and the curve $y^2 = x$ from (1, 1) to (0, 0). 5,5

SECTION—B

VI. (a) Trace the locus of the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c} = 0$, where a, b, c are positive.

(b) Obtain the equation of the surface of revolution obtained by rotating the curve $3y^2 - 2z^2 = 6$, $x = 0$ about the z-axis. 7,3

VII. (a) Find the condition that the plane $lx + my + nz = p$ may touch the paraboloids $\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = \frac{2z}{c}$.

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(b) Find the equation of the tangent planes to $2x^2 - 6y^2 + 3z^2 = 5$ which pass through the line

$$x + 9y - 3z = 0 = 3x - 3y + 6z - 5.$$

5,5

VIII. (a) Find the length of the normal chord through $P(x_1, y_1, z_1)$ of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ and prove that if it is equal to 4 PG_3, where G_3 the point in which the normal chord meets the plane XOY, then P lies on the cone

$$\frac{x^2}{a^6}(2c^2 - a^2) + \frac{y^2}{b^6}(2c^2 - b^2) + \frac{z^2}{a^4} = 0.$$

(b) The section of the enveloping cone of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ whose vertex is P by the plane $z = 0$ is a rectangular hyperbola. Find the locus of P. 6,4

IX. (a) Find the equation of the enveloping cylinder of the paraboloid $ax^2 + by^2 = 2cz$ whose generators are parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

(b) Find the equation of the surface on which the normals from the point (α, β, γ) to the elliptic paraboloid $x^2 + 2y^2 = 4z$ lies. Also name the surface. 5,5

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X. (a) Show that if the origin is the centre of a conicoid, the coefficients of the first degree terms in its equation are all zero.

(b) Reduce the equation :

$$6y^2 - 18yz - 6zx + 2xy - 9x + 5y - 5z + 2 = 0$$

to the standard form, and state the nature of the surface represented by it. 4,6