

B.A./B.Sc. 5th Semester

MATHEMATICS

Paper—II

(Number Theory)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt FIVE questions in all selecting at least TWO questions from each section.

SECTION—A

- I. (a) Prove that $2^{4n} - 1$ is divisible by 15. 2
- (b) If x and y are positive integers and $x - y$ is even, show that $x^2 - y^2$ is divisible by 4. 4
- (c) Show that square of an integer is of the form $3q$ or $3q + 1$ but not of the form $3q + 2$. 4
- II. (a) Prove that $\gcd(a, b) = \gcd(a, b + ax) = \gcd(a + by, b) \forall x, y \in \mathbb{Z}$. 5
- (b) Find integers x & y so that $12378x + 3054y = 6$. 5
- III. (a) Find general solution of the equation $91x + 221y = 1053$. 5
- (b) Show that for each prime $p \geq 5$, $p^2 + 2$ is a composite number. 5

- IV. (a) If $2^n + 1$ is an odd prime, show that n is equal to a power of 2. 5
- (b) If $a \equiv b \pmod{m}$, then prove that $a + n \equiv b + n \pmod{m}$ and $an \equiv bn \pmod{m}$ for $n \in \mathbb{Z}$. 5
- V. (a) Show that $3^{287} - 3$ is divisible by 23. 5
- (b) Solve $2x + 7y \equiv 5 \pmod{12}$. 5

SECTION—B

- VI. (a) Solve $x \equiv 5 \pmod{11}$, $x \equiv 14 \pmod{29}$, $x \equiv 15 \pmod{31}$ by Chinese Remainder Theorem. 5
- (b) Show that $a^7 \equiv a \pmod{42}$ for all $a \in \mathbb{Z}$. 5
- VII. (a) Prove that an integer $p > 1$ is a prime number iff $\lfloor p-2 \rfloor \equiv 1 \pmod{p}$. 5
- (b) Show that $\lfloor p-1 \rfloor \equiv (p-1) \pmod{(1+2+3+\dots+(p-1))}$ for any prime p . 5
- VIII. (a) For any odd prime p , show that $1^2 \cdot 3^2 \cdot 5^2 \cdot \dots (p-2)^2 \equiv (-1)^{\frac{p+1}{2}} \pmod{p}$ using Wilson's theorem. 5
- (b) If p and $2p + 1$ both are odd primes and $m = 4p$, then prove that $\phi(m + 2) = \phi(m) + 2$. 5

IX. (a) Find positive integer m so that

$$\phi(2m) = \phi(m). \quad 5$$

(b) Find the last two digits in ordinary representation of 3^{400} . 5

X. (a) If x is real number, prove that

$$[x] + [-x] = \begin{cases} 0 & , \text{ if } x \in Z \\ -1 & , \text{ if } x \notin Z. \end{cases} \quad 5$$

(b) Verify Mobius Inversion formula for $n = 24$. 5

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