

Exam. Code : 103205

Subject Code : 1190

B.A./B.Sc. 5th Semester

MATHEMATICS

Paper—II

(Number Theory)

Time Allowed—3 Hours] [Maximum Marks—50

Note :—Attempt **FIVE** questions, selecting at least **ONE** question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Prove by mathematical induction that $3^{4n+1} + 2^{2n+2}$ is divisible by 7 for every natural number n. 5
- (b) Show that the product of any m consecutive integers is divisible by $\lfloor m \rfloor$. 5

II. (a) If $c \mid a$, $c \mid b$ and $\gcd\left(\frac{a}{c}, \frac{b}{c}\right) = 1$, show that $\gcd(a, b) = c$. 5

(b) Find all the solutions in positive integers of the Diophantine equation $15x + 7y = 111$. 5

SECTION—B

III. (a) If x and y are positive real numbers, prove that $[x][y] \leq [xy]$; $[x]$, $[y]$, $[xy]$ are greatest integer functions. <https://www.gnduonline.com> 5

(b) Find the number and sum of the divisors of the natural number 10800. 5

IV. (a) For even integer n, prove that $\phi(2n) = 2\phi(n)$. ϕ is Euler's phi function. 5

(b) Verify Mobius Inversion formula for $n = 24$. 5

SECTION—C

V. (a) If $a \equiv b \pmod{m}$, then prove that $a^p \equiv b^p \pmod{m}$, $p \in \mathbb{N}$. 5

(b) Solve $13x \equiv 3 \pmod{47}$. 5

(a) Prove that the linear congruence $ax \equiv b \pmod{m}$ has a solution iff $d \mid b$ where $d = \gcd(a, m)$.

5

(b) Solve $x \equiv 1 \pmod{3}$, $x \equiv 2 \pmod{5}$, $x \equiv 3 \pmod{7}$.

5

SECTION—D

VII. (a) If $\gcd(a, 133) = \gcd(b, 133) = 1$, then using Fermat's theorem prove that $a^{18} \equiv b^{18} \pmod{133}$.

5

(b) If $a^p \equiv b^p \pmod{p}$ for any prime p , then prove that $a \equiv b \pmod{p}$.

5

VIII. (a) Show that $\sqrt{18} \equiv -1 \pmod{437}$.

5

(b) Using numerical, encipher the word 'STUDY' and decipher the word 'ERRN'.

5