

B.A./B.Sc. - 3rd Sem. (Old Syllabus 2015)

(2118)

Paper: Mathematics Paper-II

(Co-ordinate & Solid Geometry)

Time allowed: 3 hrs.

Max. Marks: 50

Note: Attempt Five questions in all, selecting at least Two questions from each section.

Section-A

1. a) If by rotating the axes through an angle θ , the expression $ax^2+2hxy+by^2$ transforms to $Ax'^2+2Hx'y'+By'^2$, show that $a+b=A+B$ and $ab-h^2=AB-H^2$.
- b) Find the angle through which the axes be rotated so that the equation $3x^2+2xy+3y^2-1=0$ is transformed into $4x^2+2y^2-1=0$. (5)
2. a) Identify the conic represent by the equation $9x^2+24xy+16y^2-2x+14y+1=0$ and transform it into another equation of the same degree without xy term. (4)
- b) Prove that the pair of lines $a^2x^2+2h(a+b)xy+b^2y^2=0$ is equally inclined to the pair of lines $ax^2+2hxy+by^2=0$. Find k so that $x^2+5xy+4y^2+3x+2y+k=0$ may represent a pair of straight lines. (5)
3. a) Show that the tangents at the extremities of a focal chord of parabola intersect at right angles on the directrix. (5)
- b) In the conic $3x^2+4y^2=5$, find the equation of the diameter which is conjugate to $3x+y=0$. (5)
4. a) Find the equation to the parabola whose focus at the point $(1,-1)$ and whose vertex is the point $(2,1)$. (5)
- b) Find the lengths of the axes, centre, eccentricity and co-ordinates of the foci of the conic $4x^2+9y^2-8x+36y+4=0$. (5)
5. a) Show that the portion of the tangent at any point of an ellipse intercepted between the point of contact and the directrix subtends a right angle at the vertex. (5)
- b) Show that the locus of the poles with respect to the parabola $y^2=4ax$ of the tangents to the hyperbola $x^2-y^2=a^2$ is the ellipse $4x^2+y^2=4a^2$. (5)

(2)

Section-B

6. a) Prove that the planes $x=cy+bz$, $y=az+cx$, $z=bx+ay$ pass through one line if $a^2+b^2+c^2+2abc=1$ and show that the line of intersection, then is $\frac{x}{\sqrt{1-a^2}} = \frac{y}{\sqrt{1-b^2}} = \frac{z}{\sqrt{1-c^2}}$ (6)
- b) Show that the following set of planes $x-z-1=0$, $x+y-2z-3=0$, $x-2y+z-3=0$ form a triangular prism. (4)
7. a) Find the equation of the plane $2x+y+3z=5$, referred to parallel axes through the point $(1,-2,3)$ (2)
- b) Find the equation of the surface $3x^2+5y^2+3z^2+2yz+2zx+2xy=1$ referred to the axes through the same origin with direction ratios $-1,0,1$; $1,-1,1$; $1,2,1$. (4)
- c) Obtain the equation of the sphere described on the join of the points $(2,-3,4)$, $(-5,6,-7)$ as diameter. (4)
8. a) Obtain the sphere having its centre on the line $5y+2z=0$, $2x-3y=0$ and passing through $(0,-2,-4)$, $(2,-1,-1)$. (4)
- b) Obtain the equation of the circle lying on the sphere $x^2+y^2+z^2-2x+4y-6z+3=0$ and having its centre at $(2,3,-4)$ (4)
- c) Show that the following set of points are concyclic:
 $(5,0,2)$, $(2,-6,0)$, $(7,-3,8)$, $(4,-9,6)$ (2)
9. a) Obtain the equation of the sphere which passes through the circle $x^2+y^2=4$, $z=0$ and is cut by the plane $x+2y+2z=0$ in a circle of radius 3. (5)
- b) Find the co-ordinates of the points where the line $\frac{x+3}{4} = \frac{y+4}{3} = \frac{z-8}{5}$ intersect the sphere $x^2+y^2+z^2+2x-10y=23$. (5)
10. a) Find the equation of the sphere which touches the sphere $x^2+y^2+z^2-x+3y+2z-3=0$ at the point $(1,1,-1)$ and pass through the origin. (4)
- b) Show that any point on radical line has constant power with respect to the three given spheres. (3)
- c) Show that power of a point (a,β,γ) with respect to sphere $x^2+y^2+z^2+24x+2\beta y+2\gamma z+d=0$ is $\alpha^2 + \beta^2 + \gamma^2 + 2u\alpha + 2\beta\beta + 2w\gamma + d$. (3)
