Exam. Code : 103203
Subject Code : 1116

B.A./B.Sc. 3rd Semester

MATHEMATICS

Paper—II (Analytical Geometry)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each section. All questions carry equal marks.

SECTION—A

1. (a) Prove that the equation :

$2x^2 + 7xy + 3y^2 + 8x + 14y + 8 = 0$ represent pair of straight lines. Also find their point of intersection and angle between them.

(b) Find angle between the lines joining the origin to the intersection of straight line $y = mx + c$ with the curve $x^2 + y^2 = a^2$. Prove that they are perpendicular if $2c^2 = a^2(1 + m^2)$.

2. (a) Show that if $ax^2 + 2hxy + by^2 = 1$ and $a'x^2 + 2h'xy + b'y^2 = 1$ represent same conic and the axes are rectangular then :

$(a - b)^2 + 4h^2 = (a' - b')^2 + 4h'^2$.

(b) Transform the equation :

$-10y + 5x^2 - 2xy + 5y^2 + 2x - 7 = 0$ to the form $AX^2 + BY^2 = C$ by suitable transformation of axes.

3. (a) Prove that the lines $2x^2 + 6xy + y^2 = 0$ are equally inclined to the lines $4x^2 + 18xy + y^2 = 0$.

(b) Find the equation of common tangent to the parabolas $y^2 = 4ax$ and $x^2 = 4by$.

4. (a) Show that the locus of points such that two of three normals from them to the parabola $y^2 = 4ax$ coincide is $27ay^2 = 4(x - 2a)^3$.

(b) Show that the locus of the poles of the tangents to the parabola $y^2 = 4ax$ w.r.t. the parabola $y^2 = 4bx$ is the parabola $y^2 = \frac{4b}{a}x$.

5. (a) Show that the locus of the middle points of chords of parabola $y^2 = 4ax$ which are of constant length $2l$ is $(4ax - y^2)(y^2 + 4a^2) = 4a^2l^2$.

(b) Prove that the locus of the foot of perpendicular from the focus on any tangent to the parabola is the tangent to the vertex.

SECTION—B

6. (a) Prove that planes $x = cy + bz$, $y = az + cx$ and $z = bx + ay$ pass through one line if :

$$a^2 + b^2 + c^2 + 2abc = 1.$$

Also find the equation of line.

(b) Show that the planes $x + y - z = 2$, $2x - y - z = -2$ and $x - 5y + z = -4$ form a triangular prism.

7. (a) Find the equation of the ellipse whose centre is the origin and passes through the points (2, 2) and (1, 4). Also find the length of major and minor axes and the eccentricity.

(b) Show that the chord of contact of perpendicular tangents to the ellipse :

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \text{ always touches the ellipse}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{a^2+b^2}.$$

8. (a) Find the locus of point of intersection of tangents to the hyperbola which include an angle α, where equation of hyperbola is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

(b) Find director circle of the hyperbola :

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$$

9. (a) Reduce the equation $x^2 - y^2 - z^2 + 6yz - 6x + 6y - 2z = 2$ to the form in which linear terms are absent.

(b) Find the equation of sphere which passes through points (1, 0, 0), (0, 1, 0) and (0, 0, 1) and has its radius as small as possible.

10. (a) Show that line joining the centres of two spheres is perpendicular to their radical planes.

(b) Find the centre and radius of circle given by $x^2 + y^2 + z^2 - 2y - 4z = 11$, $x + 2y + 2z - 15 = 0$.