Exam. Code : 103203
Subject Code : 1114

B.A./B.Sc. 3rd Semester

MATHEMATICS (Analytical Geometry)

Paper—II

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Candidates are to attempt **FIVE** questions, **ONE** from each section. **Fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Transform $5x^2 - 2xy + 5y^2 + 2x - 10y - 7 = 0$ to rectangular axes through (0,1) inclined at an angle $\frac{\pi}{4}$ to the original axes.

   (b) Prove that the lines bisecting the angle between the bisectors of the angles between the lines $ax^2 + 2hxy + by^2 = 0$ are given by $(a - b)(x^2 - y^2) + 4hxy = 0$.

2. (a) Through what angle, the axes should be rotated to remove the mixed term in the equation $5x^2 - 4xy + 5y^2 - 3x + 4y - 5 = 0$.

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(b) If p, q be the lengths of perpendiculars drawn from point (–1, 2) to the pair of straight lines given by equation $2x^2 - 5xy + 2y^2 + 3x - 3y + 1 = 0$, then prove that $pq = \frac{12}{5}$.

SECTION—B

3. (a) Show that the locus of point such that two of the three normals from them to the parabola $y^2 = 4ax$ coincide is $27ay^2 = 4(x - 2a)^3$.

   (b) If the tangents be drawn to the parabola $y^2 = 4ax$ from a point on the line $x + 4a = 0$. Show that the chord of contact of tangents will subtend a right angle at vertex.

4. (a) Show that the locus of middle point of the normal chords of the parabola $y^2 = 4ax$ is :

$$\frac{y^2}{2a} + \frac{4a^3}{y^2} = x - 2a$$

   (b) The normal at three points P, Q, R of the parabola $y^2 = 4ax$ meet in point (h,k). Prove that the centroid of triangle PQR lies on the axes at distance $\frac{2}{3}(h - 2a)$ from the vertex.

SECTION—C

5. (a) If the chord of contact of tangents from (a, β) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ touches the circle $x^2 + y^2 = c^2$, show that the point (α, β) lies on ellipse $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{c^2}$.

(b) The sum of focal distances of any point on an ellipse is constant and equal to the length of major axis, which is further greater than the distance of foci.

6. (a) If the normal at any point P of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the major axis in G, find the locus of the mid point of the chord PG.

(b) Prove that the locus of the poles of the normal chords of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{a^6}{x^2} - \frac{b^6}{y^2} = \left(a^2 + b^2\right)^2$.

SECTION—D

7. (a) A sphere of constant radius 2k passes through the origin and meets the axes in A,B,C. Show that the locus of centroid of tetrahedron OABC is the sphere $x^2 + y^2 + z^2 = k^2$.

(b) Find the coordinates of the point on the sphere $x^2 + y^2 + z^2 - 4x + 2y - 4 = 0$, the tangent planes at which are parallel to the plane $2x - y + 2z = 1$.

8. (a) Show that the sum of the squares of the intercepts made by a given sphere on any three mutually perpendicular straight lines through a fixed point is constant.

(b) Prove that the plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius unity. Also find the equation of the sphere, which has this circle as one of the great circles.