Exam. Code : 103203
Subject Code : 1119

B.A./B.Sc. Semester—III

MATHEMATICS

Paper—I (Analysis)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each section. All questions carry equal marks.

SECTION—A

I. (a) State and prove Cauchy's First Theorem on Limits.

(b) Prove that the sequence $\left\{\frac{2n-7}{3n+2}\right\}$ is (i) bounded (ii) monotonically increasing and (iii) convergent. 5,5

II. (a) Show that the sequence $\{a_n\}$ where

$$a_n = 1+\frac{1}{2}+\frac{1}{3}+\dots\dots+\frac{1}{n}$$

does not converge, by showing that it is not a Cauchy sequence.

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(b) If $\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = l$, then prove that the positive terms series $\sum_{n=1}^{\infty} a_n$ converges if $l < 1$ and diverges if $l > 1$. 5,5

III. (a) Prove that a necessary and sufficient condition for the convergence of a sequence $\{a_n\}$ of real number is that it is Cauchy sequence.

(b) Discuss the convergence of the series :

$$\sum \frac{(n!)^2}{(2n)!}x^n, \ x > 0.$$ 5,5

IV. (a) State and prove Leibnitz test for alternating series.

(b) Show that the sequence $\{x_n\}$ defined by $x_1 = \sqrt{2}$, $x_{n+1} = \sqrt{2+x_n}$ converges to 2. 5,5

V. (a) Discuss the convergence of series

$$\sum_{n=1}^{\infty} \frac{1}{x^n + x^{-n}}, \ x > 0.$$

(b) If the series $\sum a_n$ is absolutely convergent, then prove that $\sum a_n$ is convergent. Is its converse true ? 5,5

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SECTION—B

VI. (a) Show that a necessary and sufficient condition for the integrability of a bounded function f on [a, b] is that to every $\varepsilon > 0$, however small, there corresponds $\delta > 0$ such that for every partition P of [a, b] with norm $\mu(P) < \delta$, $U(P, f) - L(P, f) < \varepsilon$.

(b) Let $f(x) = 3x + 1$ on [1, 2]. Prove that f is R-integrable on [1, 2] and $\int_1^2 f(x)\,dx = \frac{11}{2}$.

5,5

VII. (a) State and prove Fundamental Theorem of Integral Calculus.

(b) Define absolutely convergent integral. Show that

$$\int_0^1 \frac{\sin\frac{1}{x}}{x^p}dx,\ p > 0 \text{ converges absolutely for } p < 1.$$

5,5

VIII. (a) If f is R-integrable on [a, b] and c is number such that a < c < b then f is R-integrable on [a, c] and [c, b]. Also show that

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

(b) Show that the improper integral $\int_a^b \frac{dx}{(x-a)^n}$ converges iff $n < 1$. 5,5

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IX. (a) Give an example of a bounded function f defined on a closed interval [a, b] such that |f| is R-integrable but f is not.

(b) Prove that $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx = \beta(m,n), m > 0, n > 0$.

5,5

X. (a) Prove that $\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ where m, n > 0.

(b) Test for convergence of the integral $\int_0^\infty \left(\frac{1}{x} - \frac{1}{\sinh x}\right)\frac{dx}{x}$. 5,5

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