Exam. Code : 103203
Subject Code : 1115

B.A./B.Sc. 3rd Semester

MATHEMATICS

Paper—I (Analysis)

Time Allowed—Three Hours] [Maximum Marks—50

Note :—Attempt **FIVE** questions in all, selecting at least **TWO** questions from each section. All questions carry equal marks.

SECTION—A

I. (a) Prove that every convergent sequence is bounded. Is its converse true ?

(b) Prove that the sequence $\{a_n\}$, where

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots\dots + \frac{1}{2n},$$

is convergent. 5,5

II. (a) Apply Cauchy's General Principle of convergence to show that $\{a_n\}$ converges, where :

$$a_n = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots\dots\dots + \frac{1}{n^2}.$$

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(b) If $\lim\limits_{n\to\infty} (a_n)^{\frac{1}{n}} = l, a_n \geq 0$, then show that the series Σa_n is convergent if $l < 1$ and divergent if $l > 1$. 5,5

III. (a) If $a_n \to l$ as $n \to \infty$, then prove that :

$$x_n = \frac{a_1 + a_2 + a_3 + \dots\dots + a_n}{n} \to l \text{ as } n \to \infty.$$

(b) Discuss the convergence or divergence of the series

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \dots\dots\dots.$$ 5,5

IV. (a) Prove that $\left\{\frac{n^3}{n^3+1}\right\}$ is Cauchy's sequence.

(b) Discuss the convergence or divergence of the series :

$$1 + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \dots\dots\dots, x > 0.$$ 5,5

V. (a) If $\{a_n\}$ is monotonically decreasing sequence of positive terms and converges to zero, then show that $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ is convergent.

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(b) Use Cauchy's Integral Test to show that the series $\sum \frac{1}{n^p}, p > 0$ converges if $p > 1$ and diverges if $p \leq 1$. 5,5

SECTION—B

VI. (a) State and prove Darboux's Theorem.

(b) Show that :

$$\int_1^2 f(x)dx = 11, \text{ where } f(x) = 4x + 5.$$ 5,5

VII. (a) Give an example of a bounded function which is not R-integrable over [a, b].

(b) Show that $\int_0^\infty \sin x^2 \, dx$ is convergent. 5,5

VIII. (a) Prove that every continuous function is Riemann integrable.

(b) Show that $\beta(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1} dx$ exists if and only if m and n are both positive. 5,5

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IX. (a) If f is R-integrable on [a, b], then | f | is also R-integrable on [a, b] and

$$\left| \int_a^b f(x)\,dx \right| \leq \int_a^b |f|(x)\,dx.$$

(b) Test the convergence of $\int_0^1 \frac{\sin \frac{1}{x}}{\sqrt{x}} dx$. 5,5

X. (a) Examine the convergence of the integral :

$$\int_0^1 \frac{1}{\sqrt{x - x^2}} dx.$$

(b) Show that $\int_0^\infty \left(\frac{1}{1+x} - e^{-x} \right) \frac{dx}{x}$ is convergent.

5,5

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