Exam. Code : 103203
Subject Code : 1107

B.A./B.Sc. 3rd Semester

MATHEMATICS

Paper—I

(Analysis)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— There are *eight* questions. Candidates are required to attempt any *five* questions. All questions carry equal marks.

SECTION—A

1. (a) Prove that a monotonically decreasing sequence is convergent if and only if it is bounded below. Moreover it converges to g.l.b. of range.

(b) Prove that sequence defined by $x_1 = \sqrt{7}, x_{n+1} = \sqrt{7 + x_n}$ is convergent and converges to positive root of $y^2 - y - 7 = 0$.

2. (a) State and prove Cauchy's second theorem on limits.

(b) If $x_1 = \frac{1}{2}\left(x + \frac{a}{x}\right), \; x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$ for all $n \in N$ where x, $a > 0$ then show that $\lim\limits_{n \to \infty} x_n = \sqrt{a}$.

https://www.gnduonline.com

SECTION—B

3. (a) Test the convergence and divergence of the series

$$1 + \frac{2}{1}\frac{1}{2} + \frac{2}{1}\frac{4}{3}\frac{1}{3} + \frac{2}{1}\frac{4}{3}\frac{6}{5}\frac{1}{4} + \ldots\ldots\ldots$$

(b) If $a_n > 0$ and $\lim\limits_{n \to \infty} \frac{a_n}{a_{n+1}} = l$ then show that series $\sum_1^\infty a_n$ is convergent if $l > 1$ and divergent if $l < 1$.

4. (a) Prove that sum of absolutely convergent series is independent of the order of terms.

~~(b) Discuss the~~ convergence of the series

$$1 + \frac{2x}{3^2} + \frac{3^2 x^2}{3!} + \frac{4^3 x^3}{4!} + \ldots\ldots\ldots\ldots\infty, x > 0$$

SECTION—C

5. (a) Prove that every monotonic function on [a, b] is Riemann integrable on [a, b].

(b) If $f(x) = \begin{cases} \cos x \text{ when } x \text{ is rational} \\ \sin x \text{ otherwise} \end{cases} \; 0 \le x \le \pi/4$,

Is f Riemann integrable on [0, π/4] ?

https://www.gnduonline.com

https://www.gnduonline.com
https://www.gnduonline.com

6. (a) Prove that if $f(x) = 2x+3$, $0 \le x \le 2$ then f is Riemann integrable

(b) Prove that if f is bounded function on [a, b] then $f \in R[a, b]$ iff for given $\epsilon > 0$ there exist a partition P of [a, b] that $U(P, f) - L(P, f) < \epsilon$.

SECTION—D

7. (a) Evaluate :

$$\int_0^\infty \frac{x^2}{1+x^4}\,dx.$$

(b) Test the convergence of $\int_0^\pi \frac{\sqrt{x}}{\sin x}\,dx$.

8. (a) Prove that $\beta(l,m) = \dfrac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$

(b) Show that :

$$\int_0^\pi x^m (\csc x)^n\,dx \text{ exist if and only if } n < m + 1$$

https://www.gnduonline.com

6. (a) Prove that if $f(x) = 2x+3$, $0 \le x \le 2$ then f is Riemann integrable

(b) Prove that if f is bounded function on [a, b] then $f \in R[a, b]$ iff for given $\epsilon > 0$ there exist a partition P of [a, b] that $U(P, f) - L(P, f) < \epsilon$.

SECTION—D

7. (a) Evaluate :

$$\int_0^\infty \frac{x^2}{1+x^4}\,dx.$$

(b) Test the convergence of $\int_0^\pi \frac{\sqrt{x}}{\sin x}\,dx$.

8. (a) Prove that $\beta(l,m) = \dfrac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$

(b) Show that :

$$\int_0^\pi x^m (\csc x)^n\,dx \text{ exist if and only if } n < m + 1$$

140(2121)/MM-2008 3 10000

https://www.gnduonline.com
https://www.gnduonline.com
https://www.gnduonline.com