Exam. Code : 103202

Sub. Code : 1048

B.A./B.Sc. 2nd Semester
MATHEMATICS
-I
(Calculus & Differential Equations)

Time : 3 Hours] **[Max. Marks : 50**

Note :- Attempt FIVE questions in all, selecting at least TWO questions each from section.

SECTION-A

1. (a) Find the equation of the cubic which has the same asymptotes as the curve $x^2y - xy^2 + xy + y^2 + x - y = 0$ and passes through the points (0, 0), (0, 1) and (1, 0).

   (b) Determine a and b so that the curve $y = ax^2 + 3bx^2$ has a point of inflexion at (-1, -2).

5,5

2. (a) Find the position and nature of the double points on the curve : :

$$2(x^3 + y^3) — 3(3x^2 + y^2) + 12x - 4 = 0.$$

   (b) If p_1, p_2 are the radii of the curvature at the extremities of a focal chord of a parabola whose semi-latus rectum is l prove that :

$$(\rho_1)^{-\frac{2}{3}} + (p_2)^{-\frac{2}{3}} = (l)^{-\frac{2}{3}}.$$

5,5

3. (a) Trace the curve $y^2 = x(x - a)^2$, a> 0.

(b) Evaluate :

$$\int \frac{dx}{a + b \tanh x'}, \; a \neq b \; .$$ 5,5

4. (a) Obtain a reduction formula for $\int \sec^{2n+1} x dx$.

Hence evaluate $\int \sec^5 x dx$

(b) If 0 <e < 1, prove that :

$$\frac{2}{\pi} \int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{1 - e^2 \sin^2 x}} = 1 + \frac{1^2}{2^2} e^2 + \frac{1^2 . 3^2}{2^2 . 4^2} e^4 + \frac{1^2 . 3^2 . 5^2}{2^2 . 4^2 . 6^2} e^6 + ...$$ 5,5

5. (a) Find the area common to the circle $x^2 + y^2 = 4$ and the ellipse $x^2 + 4y^2 = 9$.

(b) Prove that :

$$\int_0^1 \frac{\log (1 + x)}{1 + x^2} dx = \frac{\pi}{8} \log 2.$$ 5,5

SECTION-B

6. (a) Solve :

$$y(xy + 2x^2y^2)dx + x(xy — x^2y^2)dy = 0.$$

(b) Solve :

$$p^3 + 8y^2 = 4pxy.$$ 5,5

7. (a) Find the general and singular solution of the equation $xp^2 - (x - a)^2 = 0$.

(b) Prove that the system of confocal and coaxial parabolas $y^2 = 4a(x + a)$ is self orthogonal. 5,5

8. (a) Solve :

$$(D^2 + 1)y = xe^x \sin 2x.$$

(b) Use method of reduction of order to solve :

$$(D^2 + 1)y = \text{cosec } x.$$ 5,5

9. (a) Solve:

$$4x \frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = 0 \text{ in series.}$$

(b) Solve in series :

$$\frac{d^2y}{dx^2} + x\ y = 0.$$ 5,5

10. (a) Solve Legendre's equation :

$$(1 - x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + n(n + 1)y = 0.$$

(b) Solve :

$$(x^2D^2 + 3xD + 1)y = \frac{1}{(1 - x)^2}.$$ 5,5