Exam. Code : 211002
Subject Code : 5540

M.Sc. (Mathematics) 2nd Semester

REAL ANALYSIS—II

Paper—MATH-561

Time Allowed—Three Hours] [Maximum Marks—100

Note :— Attempt any **TWO** questions from each unit. Each question carries equal marks.

UNIT—I

1. State and prove Arzela's theorem.
2. Suppose K is compact and $\{f_n\}$ is a sequence of continuous functions on K and $\{f_n\}$ converges pointwise to a continuous function f on K. Also, $f_n(x) \geq f_{n+1}(x)$, $\forall\ x \in K$, n = 1, 2, 3, ... Then $f_n \to f$ uniformly on K.
3. The sequence of functions $\{f_n\}$ defined on E, converges uniformly on E if and only if for every $\varepsilon > 0$ there exists an integer N such that $m \geq N$, $n \geq N$, $x \in E$ implies $| f_n(x) - f_m(x) | \leq \varepsilon$.
4. Define equicontinuity. If K is compact and $f_n \in \mathcal{C}(K)$ for n = 1, 2, 3, ... and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then $\{f_n\}$ is uniformly bounded on K and contains a uniformly convergent subsequence.

UNIT—II

5. Define a measurable set. Prove that outer measure of an interval is its length.
6. If m is a countably additive, translation invariant measure defined on a σ-algebra containing the set P, then m[0, 1) is either zero or infinity.
7. If A is countable then show that $m^*A = 0$.
8. Show that the interval (a, ∞) is measurable.

UNIT—III

9. Define a measurable function. Let c be a constant and f and g be two real valued measurable functions defined on the same domain, then $f \pm g$, $f + c$ and cf are also measurable.
10. Define a characteristic function and a simple function. Prove that $\chi_{A \cap B} = \chi_A \cdot \chi_B$ and $\chi_{\bar{A}} = 1 - \chi_A$.
11. Define almost everywhere. If f is measurable function and f = g a.e., then g is measurable.
12. State and prove Egoroff's theorem.

UNIT—IV

13. Give an example of a function which is Lebesgue integrable but not Riemann integrable.
14. State and prove monotone convergence theorem.
15. State and prove bounded convergence theorem.
16. Let f be a non-negative measurable function. Show that $\int f = 0$ implies f = 0 a.e.

UNIT—V

17. State and prove Vitali's lemma.

18. If f is integrable on [a, b] and $\int_a^x f(t)\,dt = 0$, for all $x \in [a, b]$, then f(t) = 0 a.e. in [a, b].

19. Define absolute continuity. Show that every absolutely continuous function is the indefinite integral of its derivative.

20. Let f be an increasing real valued function on the interval [a, b]. Then f is differentiable almost everywhere. The derivative f′ is measurable and

$$\int_a^b f'(x)\,dx \le f(b) - f(a).$$