

Exam. Code : 211002

Subject Code : 5542

M.Sc. (Mathematics) 2nd Semester

ALGEBRA-II

Paper-MATH-563

Time Allowed—3 Hours]

[Maximum Marks—100

Note :- The candidates are required to attempt two questions from each unit. Each question carries equal marks.

UNIT-I

1. (a) Prove that an irreducible element in a PID is always prime. 5
(b) Prove that every Euclidean Domain is a PID. 5
2. (a) Prove that $F[x]$, F field, is an Euclidean ring. 5
(b) If R is an integral domain with unit element, then prove that any unit in $R[x]$ must be unit in R . 5
3. (a) Is $\mathbb{Z}[x]$ a Principal Ideal Domain ? Justify your answer. 5
(b) Prove in UFD, two non-zero elements possess HCF. 5
4. Prove that if a ring R is PID then it is UFD. Is the converse true ? Justify. 10

UNIT-II

5. (a) If $a, b \in K$ are algebraic over F such that $[F(a) : F] = m$ and $[F(b) : F] = n$ and $\gcd(m, n) = 1$, then prove that $[F(a, b) : F] = mn$. 5
(b) Prove that $\mathbb{Q}(\sqrt{2}, \sqrt[3]{5})$ is a simple extension. 5
6. (a) Give an example of an algebraic extension of a field which is not finite. 5
(b) For a prime p , find the degree of splitting field of $x^p - 1$ over \mathbb{Q} . 5
7. (a) If K is algebraic over E and E is algebraic over F , then prove that K is algebraic over F . 5
(b) Prove that the splitting field of a polynomial over F is unique upto F -isomorphism. 5
8. (a) Let θ be a root of an irreducible polynomial $x^3 - 2x - 2$. Then find $\frac{1 + \theta}{1 + \theta + \theta^2}$ in $\mathbb{Q}(\theta)$. 5
(b) Let $f(x)$ be a non-constant polynomial over field F , then prove that there exists an extension E of F in which $f(x)$ has a root. 5

UNIT-III

9. Prove that a regular n -gon is constructible if and only if $\phi(n)$ is a power of 2. 10
10. (a) Prove that the characteristic of a finite field F is prime number say p and F contains a subfield isomorphic to \mathbb{Z}_p . 5
(b) Construct a field with 27 elements. 5

11. (a) Prove that the multiplicative group of non-zero elements of a finite field is cyclic. 5
 (b) Show that all the roots of an irreducible polynomial over finite field are distinct. 5
 12. Prove that a finite separable extension of a field is simple. 10

UNIT-IV

13. Prove that for a Galois extension E/F , there is 1-1 correspondence between the subgroups of $G(E/F)$ and the subfields of E containing F . 10
 14. Let $E = \mathbb{Q}(\sqrt[3]{2}, w)$, where $w^3 = 1, w \neq 1$. Let $H \subseteq G(E/\mathbb{Q})$ and $H = \{\sigma_1, \sigma_2\}$ where σ_1 is the identity map and $\sigma_2(\sqrt[3]{2}) = \sqrt[3]{2}w$ and $\sigma_2(w) = w^2$. Find E_H . 10
 15. Suppose that the Galois group $G(E/F)$ of a polynomial $f(x)$ over F is a solvable group, prove that E is solvable by radicals over F . 10
 16. (a) Give an example each of a polynomial which is solvable by radicals and a polynomial which is not solvable by radicals. 7
 (b) Express $x_1^3 + x_2^3 + x_3^3$ as a rational function of elementary symmetric function. 3

UNIT-V

17. State fundamental theorem of finitely generated module over PID. Prove that a finitely generated torsion-free module over PID is free. 10
 18. State and prove Schur's lemma for simple modules. 10

19. Prove that over PID, a submodule of finitely generated module is finitely generated. Is the result true in general? Justify. 10
 20. Let R be a commutative ring with unity and M, N free R -modules. Prove that $\text{Hom}_R(M, N)$ is a free R -module if M is finitely generated. Further if N is also finitely generated, then find the basis of $\text{Hom}_R(M, N)$. 10

<http://www.gnduonline.com>

Whatsapp @ 9300930012

Your old paper & get 10/-

पुराने पेपर्स भेजे और 10 रुपये पायें,

Paytm or Google Pay से