Exam. Code : 211001

Subject Code : 3834

M.Sc. Mathematics 1ˢᵗ Semester

REAL ANALYSIS—I

Paper : MATH-551

Time Allowed—3 Hours] [Maximum Marks—100

Note :— (1) Attempt **five** questions in all, selecting at least **one** question from each part.

(2) Include the necessary results, depending upon the credit of the question.

(3) In the following, X denotes a metric space with metric d.

PART—I

1. (a) Prove that finite Cartesian product of countable sets is countable. 7

   (b) Prove that open balls/neighborhoods in X are open subsets of X. 6

   (c) Let $E \subset X$ and x be a limit point of E. Prove that for every $\epsilon > 0$, the set $B(x ; \epsilon) \cap E$ is infinite. 7

2. (a) Prove that every closed and bounded subset of R is compact. Is it true for arbitrary metric spaces ? 8

   (b) Prove that every infinite compact metric space has a limit point. 6

   (c) Prove that there are uncountably many irrational numbers inside the Cantor set. 6

PART—II

3. (a) Prove that [0, 1] is a connected subset of reals. 6

   (b) Let A and B be separated subsets of X such that $A \cup B = X$. Prove that both A and B are open as well as closed in X. 6

   (c) Prove that the sum as well as the product of two functions of bounded variation are functions of bounded variation. 8

4. (a) Does there exist a sequence which has uncountably many convergent subsequences ? Justify your answer. 6

   (b) Let $\{x_n\}$ be a Cauchy sequence in X, containing a convergent subsequence. Prove that $\{x_n\}$ is a convergent sequence. 7

   (c) State and prove the nested interval property of reals. 7

PART—III

5. (a) State and prove the Banach contraction principle. Also show that the completeness of the domain is not redundant. **8**

(b) Prove that the set of all irrational numbers is not a countable union of closed subsets of reals.

6

(c) Prove that continuous image of compact sets is compact. **6**

6. (a) Let $f : \mathbb{R} \to \mathbb{R}$ be a monotone function discontinuous at some $d \in \mathbb{R}$. Prove that d is a jump discontinuity of f. **10**

(b) Prove that the composition of two uniformly continuous functions, if possible, is uniformly continuous. **10**

PART—IV

7. (a) State and prove the well known inequality consisting of the upper and lower Riemann integral of a bounded function. **10**

(b) Prove that every continuous function has an antiderivative. **10**

8. (a) Prove that every monotone real function on [0, 1] is Riemann-Stieltje integrable. **10**

(b) State and prove the well known *integration by parts* formula for the Riemann integral. **10**