Exam. Code : 103203
Subject Code : 1122

B.A./B.Sc. 3rd Semester

MATHEMATICS

Paper—I : (Analysis)

Time Allowed—3 Hours] [Maximum Marks—50

SECTION—A

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** questions from each Section. All questions carry equal marks.

I. (a) If $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n}=l$, where $|l| < 1$, then show that $\lim\limits_{n\to\infty} a_n = 0$.

(b) Show that

$$\lim_{n\to\infty}\left[\frac{1}{n^2}+\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\frac{1}{(n+3)^2}....+\frac{1}{(2n)^2}\right]=0$$

5,5

II. (a) Prove that every sequence contains a monotonic subsequence.

(b) Using Cauchy's Integral Test, discuss the convergence or divergence of the series $\sum\limits_{n=1}^{\infty}\frac{1}{n^p}$, $p>0$. 5,5

http://www.gnduonline.com

III. (a) If $a_n = 1+\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{n}-\log n$, then prove that $\{a_n\}$ is a monotonically decreasing sequence. Prove that it is convergent.

(b) Test the series $1+\frac{\alpha\beta}{1\cdot\gamma}x+\frac{\alpha(\alpha+1)\beta(\beta+1)}{1.2\cdot\gamma\cdot(\gamma+1)}x^2+....\infty$, $x>0$ for convergence. 5,5

IV. (a) If $\lim\limits_{n\to\infty}(a_n)^{\frac{1}{n}}=l$, $a_n \geq 0$, then show that series $\sum\limits_{n=1}^{\infty}a_n$ is convergent if $l<1$, and divergent if $l>1$.

(b) Discuss the convergence or divergence of the series :

$$x+\frac{2^2x^2}{2!}+\frac{2^3x^3}{3!}+\frac{2^4x^4}{4!}+......,\ x>0.$$ 5,5

V. (a) If $\{a_n\}$ is monotonically decreasing sequence of positive terms and converges to zero, then prove that $\sum\limits_{n=1}^{\infty}(-1)^{n-1}a_n$ is convergent.

(b) Prove that $\lim\limits_{n\to\infty}\left[\left(\frac{2}{1}\right)^1\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3...\left(\frac{n+1}{n}\right)^n\right]^{\frac{1}{n}}=e.$

5,5

SECTION—B

VI. (a) If f is a bounded function defined on [a, b], then to every $\varepsilon > 0$, however small, there corresponds $\delta > 0$ such that

(i) $L(P,f) > \int_a^b f(x)\,dx - \varepsilon$

(ii) $U(P,f) < \int_a^{\bar{b}} f(x)\,dx + \varepsilon$ for every partition P of [a, b] with norm $\mu(P) < \delta$.

(b) If is continuous on [a, b], then prove that $f \in R\,[a, b]$. 5,5

VII. (a) If f is bounded and integrable on [a, b], then show that | f | is also bounded and integrable on [a, b] and $\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx$.

(b) Examine the convergence of $\int_0^\infty \frac{\sin x}{x}\,dx$. 5,5

VIII. (a) If f is a continuous function defined on [a, b], then show that there exists $c \in [a, b]$ such that $\int_a^b f(x)dx = f(c)(b-a)$.

(b) Show that $\int_0^\infty \left(\frac{1}{1+x} - e^{-x}\right)\frac{dx}{x}$ is convergent. 5,5

IX. (a) If f is continuous in [a, b], then show that $F(x) = \int_a^x f(t)\,dt$ is derivable on [a, b] and $F'(x) = f(x)\ \forall x \in [a,b]$.

(b) Prove that $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\,dx = \beta(m, n), m > 0, n > 0.$

5,5

X. (a) Prove that $\frac{\Gamma(m)\,\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(2m)} = \frac{\sqrt{\pi}}{2^{m-1}}$, where m is +ve.

(b) If a > 0, then prove that $\int_a^\infty \frac{dx}{x^n}$ converges iff $n > 1$.

5,5