Exam. Code : 103201
Subject Code : 1028

B.A./B.Sc. 1st Semester

MATHEMATICS

Paper—I (Algebra)

Time Allowed—Three Hours] [Maximum Marks—50

Note :—Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. All questions carry equal marks.

SECTION—A

1. (a) Define the rank of a matrix. Compute the rank of the matrix $A = \begin{bmatrix} 1 & 5 & 3 \\ -1 & 3 & 5 \\ 1 & 0 & -2 \end{bmatrix}$ by reducing it to an equivalent matrix of the form $PAQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$.

   (b) Find the row rank of the matrix :

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}.$$

2. (a) State condition under which a set of homogeneous equations possess a (i) trivial solution or (ii) non-trivial solution, why ?

   (b) If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$, then show that the system AX = B is consistent if and only if $b_3 - 2b_1 + b_2 = 0$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

SECTION—B

3. (a) Prove that any two characteristic vectors corresponding to two distinct characteristic roots of a unitary matrix are orthogonal.

   (b) Determine eigen values and the corresponding eigen-vectors for the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

4. (a) Verify Cayley Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 0 \\ \sqrt{2} & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ and find A^{-1}.

(b) Write down the quadratic form corresponding to the symmetric matrix :

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix}.$$

SECTION—C

5. (a) Define congruent matrices and explain its fundamental properties.

(b) Show that if A is any n-rowed non-zero symmetric matrix of rank r over a field F, then there exists an n-rowed non-singular matrix P over F, such that $P'AP = \begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix}$, where A_1 is a non-singular r-rowed diagonal matrix over F and each O, is a zero matrix of the appropriate type.

6. (a) Reduce the following to canonical form and find the rank and index :

$$x^2 - 2y^2 + 3z^2 - 4yz + 5zx.$$

(b) Show that the form $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_2x_3 - 2x_3x_1 + 2x_1x_2$ is indefinite and find two set of values of x_1, x_2, x_3 for which the form assumes positive and negative values.

SECTION—D

7. (a) If α, β, γ are the roots of the equation $2x^3 - 6x^2 + 3x + k = 0$ such that $\alpha = 2(\beta + \gamma)$, find k and solve the equation.

(b) If α, β, γ, δ are the roots of the equation $x^4 + ax^2 + bx + c = 0$ find the value of

$$\sum \frac{\beta + \gamma - \delta - \alpha}{2\alpha^2}$$

8. (a) If $q > 0$, $r > 0$ then prove that the cubic $x^3 + qx + r = 0$ has one negative and two imaginary roots.

(b) Solve by Ferrari's method :

$$x^4 - 2x^3 - 5x^2 + 10x - 3 = 0.$$